manuscript No.
(will be inserted by the editor)

Shape Modeling and Understanding at the University of Genova

Geometry & Graphics Group

L. De Floriani · P. Magillo · E. Puppo · D. Canino · N. De Giorgis · R. Fellegara · U. Fugacci · F. Iuricich · G. Picciau · L. Rocca

1 Introduction

The Geometry & Graphics Group (G^3) at the University of Genova has been active in the fields of geometric modeling, computer graphics, shape analysis, spatial data structures since 1990. The research of the group focuses on techniques for representing, manipulating, visualizing and reasoning on spatial objects. Recently, the emphasis has been on structural and geometric modeling of 3D shapes, terrains and volume data. The reference applications are in scientific visualization, data analysis, geographic data processing, computer aided design and animation. The research on complex shape modeling focuses on developing efficient representations for geometric shapes. A special emphasis is dedicated on developing topological data structures and spatial indexes to efficiently encode geometry meshes, like cell and simplicial complexes, quad and hexahedral meshes of arbitrary complexity and in arbitrary dimensions, giving rise to the development of software tools made available in the public domain (see Section 2). The research in geometric modeling concentrates on quad-based representations (see Section 3) for applications to Computer Aided Design and character animation. The research on topological shape analysis is based on applying topological tools to the description and to the understanding of shapes, and specifically on morphological representation of shapes endowed with scalar fields for scientific data visualization and analysis and on computation of topologically invariants providing global quantitative and qualitative information about a shape (see Section 4). Moreover, our research studies techniques to describe shapes through a scale-space approach and to the application of such techniques in terrain analysis and human face recognition (see Section 5).

Department of Computer Science, Bioengineering, Robotics, and Systems Engineering - (DIBRIS), University of Genova, Italy

2 Data Structures for Discretized Shapes in Arbitrary Dimensions

In the applications, we always deal with shapes in two, three or higher dimensions discretized as complexes of polyhedral cells or made of simplices. Triangle and quad meshes are used for describing the boundary of 3D shapes, tetrahedral and hexahedral meshes for discretizing their volume. Simplicial complexes, the higher dimensional and non-manifold analogue of triangle and tetrahedral meshes, are used to encode the relation between subsets of points and, thus, are the basic representation of shapes for geometry understanding in higher dimensions.

In our research we focused on two approaches, one based on the development of several data structures for simplicial complexes in 2D, 3D and arbitrary dimensions, which has led to the design and the implementation of the *Mangrove TDS library* which allows the efficient management of several topological data structures for cell and simplicial complexes arbitrary dimensions and to the design and the implementation of a framework, based on spatial indexes, which can efficiently compute topological relations among the cells or simplices of the complex without explicitly encoding them.

2.1 Connectivity-based Data Structures: the Mangrove Library

Our research has focused on the development of data structures for simplicial complexes in arbitrary , which differ in the simplices and in the topological relations they encode. We have developed data structures encoding all the simplexes in a complex plus they incidence relations [13,14] as well as data structures encoding only the top simplexes plus their adjacency relations [4,15,16] These latter are more compact, but they do not allow attaching attributes to simplices not explicitly represented. Based on this work, we have designed and developed a framework, the *Mangrove Topological Data Structure (Mangrove TDS) Library* which is a C++ tool for the fast prototyping of topological data structures representing cell and simplicial complexes of any dimension, not necessarily embedded in the Euclidean space. It contains the implementation of seven data structures for cell and simplicial complexes together with operators for navigating and querying the complex based on them. Editing operators, like simplex collapse, are available and we have implemented also a new set of Euler operators on cell complexes [9], which preserve the simplicial homology, like Betti numbers and homology generators. The Mangrove TDS library ($http://mangrovetds.sourceforge.net$) is the basis for our research on topological shape analysis based on homology, as discussed in Section 4.2.

2.2 A Spatio-Topological Approach to Shape Representation

We have designed and implemented a framework based on topological spatial indexes that leads to a new data structure, the *PR-star tree* [30], in which local topological connectivity of a simplicial or cell complex is obtained through spatial locality. In contrast to topological data structures, which have focused on adjacencies or incidences, we use a spatial data structure on the complex embedding space to locally reconstruct the optimal application-dependent topological representation at runtime using the sorted geometry available from the spatial index.

The innovative feature of our approach is in computing topology through space: local spatial sorting allows the efficient reconstruction of the local mesh connectivity. Although this increases the cost of a single operation due to the construction of the local data structure, this cost is amortized over multiple accesses to elements within the same region. Moreover, by recovering the memory associated with each local data structure after the processing of that part of the mesh has completed, we achieve significant memory savings with respect to global topological data structures. As an application of PR-star, we have designed and implemented a framework [11,31] which can efficiently compute and extract morphological features in 2D and 3D scalar fields, based on Forman theory (see Section 4).

3 Geometry processing

Discrete representations of solid objects, in particular surfaces and volumes, are ubiquitous in the applications. Simplicial representations (e.g., triangular and tetrahedral meshes) have excellent mathematical properties and they are easily obtained from automatic methods for reconstruction and meshing, but most often they are irregular and unstructured. Conversely, in many applications (e.g., CAD, FEM, animation) models based on quadrangular and hexahedral cells are preferred, and often indispensable. Such models must be as regular as possible and endow in their connectivity the underlying structure of the objects represented. Obtaining such models in an automatic, or even semi-automatic way, with the quality necessary to the production pipeline, is still an open problem, in spite of at least a decade of work by several groups of researchers [2]. Our group has contributed in several ways to advance the state-of-the-art in this direction during the last five years.

In [28] we first proposed a method to convert a triangle mesh into a simplified quad mesh with faces that are as close as possible to squares of uniform size; we also extended this method to adaptive quad meshing in [3]. The same approach was extended in [25] to support modeling with subdivision surfaces of existing shapes by a reverse engineering approach. In spite of achieving results of unprecedented quality, these works failed to address two important criteria: the alignment of quad elements to shape features, such as curvature; and the construction of a semi-regular and coarse mesh.

In [29] we addressed both criteria, developing one of the first methods that is able to cover a given surface with a coarse layout of quadrangular domains, roughly aligned to the principal directions of curvature. With this work, we started to investigate the intimate mathematical relations between directional fields defined on a given surface and quadrangular meshes laid over it. Broadly speaking, N-symmetric fields are generalizations of vector fields on Riemannian manifolds, which are identical upon rotations of $2\pi/N$ about the surface normal. For instance, principal directions of curvature on a smooth surface define two mutually orthogonal 2-fields (a.k.a. *line fields*), while quadrangulations are directly connected with 4-fields (a.k.a. *cross fileds*), which are defined at each point by a cross of four vectors of the same size. A quad mesh can be viewed as a discrete version of a cross field, and its quality is determined by the *differential topology* of such a field, i.e., by the distribution of its *singularities* and of the *separatrices* connecting them. In [29] we studied how separatrices can be disentangled to obtain a coarse quad layout. Later on, we developed an approach based on cross fields to obtain quad meshes that are aware of the symmetries of objects, with applications to objects with intrinsic bilateral symmetries like humans and animals [24]. In [19], we developed another approach based on cross fields to obtain quad meshes that adapt to given animation sequences, obtained with body scanners able to capture living creatures in motion. Lately, in [26] we have generalized cross fields to *frame fields,* by incorporating anisotropy and skewness, thus making this modeling framework much more flexible. We have applied this framework to the design of quad meshes over given surfaces, also supporting user interaction to achieve results with a quality comparable to meshes obtained by expert users with long and tedious manual editing. Our latest efforts are towards the integration of fast automatic tools together with user interaction, so that an artist can maintain full control on design while delegating to the system most tedious operations. In [18], we propose a method for computing a coarse quad layout for articulated objects, which follows the intrinsic object structure described by its curve skeleton. This quad layout contains few irregular vertices of low degree: it can be refined immediately into a semi-regular quad mesh; it provides a structured domain for UV-mapping and parametrization; it can be taken as a control grid for subsequent spline modeling; and hexahedral mesh filling the object volume can be easily generated, too. Our method is fast, one-click and it does not require any parameter setting. It can be easily integrated with interactive techniques, both during the construction of the quad layout, and later on in order to proceed to model refinement. In [20], we propose a method for assisting users during editing sessions to manually design a coarse quad layout over a given surface. By analyzing existing quad meshes designed by artists, our algorithm learns the most frequently used quadrangulation patterns, which are compressed and stored in a database. Real-time queries extract patterns that satisfy user-specified constraints, enabling reuse of them during an interactive retopology session.

4 Computational Topology for Shape Analysis and Visualization

Spatially-related digital data are being produced at a constantly increasing pace and their availability is changing the approach to science and its applications. The complexity of the data derives not only from the size of currently available data sets, but also comes from the need to filter out relevant information from huge quantities of unimportant details. This leads to the need for computational tools that can efficiently process large sets of data and generate synthetic descriptors, which should adapt to different applications, such as classification, recognition, visualization, reconstruction, etc. Topology deals with qualitative geometric information, and this provides either an alternative, or at least a complementary, way to describe shapes. Advantages of topological data analysis are the robustness of topological invariants, the fact that to topology studies spaces in a coordinate-free manner and the compactness of topology-shape descriptions. The main issues in extensively using topological tools in the applications is their high computational and storage costs, and lack of scalability with the increase of dimension. In our research we have developed effective topological descriptions of discrete shapes endowed with a scalar field by using tools rooted in Morse theory [22], and we apply homological information to the analysis and understanding of shapes in medium and high dimensions.

4.1 Computation and Hierarchical Representation of Morse Complexes

Through Morse theory [22], the topology of a manifold shape M can be studied in relation with the critical points of a scalar (real-valued) function defined on M. Morse theory offers a natural and intuitive way of analyzing the structure of a scalar field as well as of compactly representing the scalar field through a decomposition of its domain into meaningful regions associated with the critical points of the field. In the application domain Forman theory [17] provides a discrete setting in which the main results from smooth Morse theory are extended to cell complexes.

We are interested in two issues related to morphological representation of scalar fields. The first one is concerned with structural problems in Morse and Morse-Smale complexes, like over-segmentation in the presence of noise, the second concerns efficiency issues arising from the very large size of the input data sets. These problems can be faced and solved by defining simplification operators on such complexes and on their morphological representations. In Morse theory an operator, called *cancellation*, has been defined which removes two critical points by locally modifying the integral lines originating and converging in those two points [21]. Cancellation, however, may increase the number of mutual incidences among cells of a Morse-Smale complex when applied on a complex in dimension higher than two. For this reason we have defined in [7], two dimension-independent simplification operators for Morse complexes, alongside with the inverse refinement operators defined as the undo of the simplification operators. These new operators, called $removal_{i,i+1}$ and $removal_{i,i-1}$, constantly reduce the number of cells in the Morse and Morse-Smale complexes. We have introduced also the updates imposed by the two simplification operators on the combinatorial structure of the Morse-Smale complex represented through a combinatorial structure the Morse Incidence Graph (MIG) and we have compared their behavior with the cancellation operator defined by Morse theory [8, 10].

The *removal* operators and their inverse, refinement operators have been used in [5] for the definition of a multi-resolution model consisting of a hierarchy of combinatorial representations of descending and ascending Morse complexes in the form of an incidence graph (MIG). Such multi-resolution model, called *Multi-resolution Morse Incidence Graph* ($MMIG$), is built starting from the ascending and descending Morse complexes, computed on the input scalar field, performing a sequence of *removal* operators. The model encodes the whole simplification history as well as a dependency relation among all the simplifications performed. From an $MMIG$ it is thus possible to dynamically extract representations of the topology of an n-dimensional scalar field in terms of the Morse complexes, at *uniform* and *variable* resolutions, by applying a sequence of refinement modifications.

4.2 Homology computation on high-dimensional data

Homology is one of the most important tools to obtain topological invariants. Homology provides global quantitative and qualitative information about a shape, such as the number of its connected components, and the number of holes and tunnels. Also, topological features are especially important in high dimensional data analysis, where pure geometric tools are usually not sufficient. The existing literature about homology computation focus on the computation of simplicial homology with coefficients in $\mathbb{Z}_2$. This topological invariant is simpler to be computed than the classical one but it fails in providing the torsion part of the homological information, present, for instance, in a Klein bottle. The classical way to compute homology with respect to any Abelian group is the *Smith Normal Form reduction (SNF)* [23]. This algorithm is based on the reduction of the boundary maps described as matrices through an approach similar to Gauss reduction. The time complexity of the SNF algorithm is super-cubical in the number of the simplices of the complex, and thus this approach is not feasible for any practical application.

Our research group has developed two different methods to improve efficiency and effectiveness of homology computation: a decomposition-based method and a hierarchical one. The decomposition strategy is based on a divide-and-conquer approach and allow us to obtain the homology with coefficients in $\mathbb{Z}$ of a complex by combining the homological information of its subcomplexes. The proposed method is called *constructive Mayer-Vietoris (MV) algorithm* [1] and is based on the *manifold-connected decomposition* introduced in [12]). Our results show that the MV algorithm requires at least 55% less space than the SNF algorithm also providing a considerable speed up (1.6 times faster than the SNF algoritm).

An interesting tool both to encode a cell complex and to compute its homology groups is the *Hierarchical Cell Complex (HCC)* [6]. An HCC implicitly encodes a virtually continuous set of complexes obtained from the original complex through the application of homology-preserving operators, which are part of a minimally complete set of operators for manipulating complexes in arbitrary dimensions [9]. We have proposed an algorithm which computes homology and homology generators on the coarsest representation of the original complex propagating them to complexes at any intermediate resolution by traversing the HCC. The computation of the homology generators on the coarsest complex of the model and the refinement of the generators at full resolution take much less time than applying the SNF algorithm directly on the original complex. Moreover, we obtain an additional reduction of computing time (between 15% and 30%) by refining the complex only in the proximity of the generators. We are currently extending this approach to o simplicial complexes.

5 Multi-scale Shape Analysis

The deep structure of scale-space of a signal refers to tracking across scales the zero-crossings of differential invariants. Classical methods for feature tracking are prone to noise and tracking errors and they provide just a coarse representation of the deep structure. In [27], we have proposed a new approach that allows us to construct a virtually continuous scale-space for scalar functions, supporting reliable tracking and a fine representation of the deep structure of their critical points. Our approach is based on a piecewise-linear approximation of the scale-space, in both space and scale dimensions. Tracking of critical points is continuous and exact in the context of such an approximation. Preliminary results were presented with applications to terrain data and range images. The main benefit of this approach is that it allows to rank the importance of critical points with respect to their persistence in the scale-space and also to measure their importance and to select the best scale at which they correspond to representative features. We are currently working on applications of this method to the automated placement of *spot heights* on relief maps, a problem in cartography that so far has been addressed just manually; and to the automatic detection of fiducial points from range maps of human faces, a challenging problem that is relevant to face recognition. We are also working on an extension of the method to work on full 3D data, such as mesh representations of solid objects, which may find several other applications.

References

1. Boltcheva, D., Canino, D., Aceituno, S.M., Léon, J.C., Floriani, L.D., Hétroy, F.: An iterative algorithm for homology computation on simplicial shapes. Computer-Aided Design **43**(11), 1457 – 1467 (2011)
2. Bommes, D., Lévy, B., Pietroni, N., Puppo, E., Silva, C., Tarini, M., Zorin, D.: Quad-mesh generation and processing: A survey. Comput. Graph. Forum **32**, 51–76 (2013)
3. Bozzo, A., Panozzo, D., Puppo, E., Pietroni, N., Rocca, L.: Adaptive quad mesh simplification. In: Eurographics Italian Chapter Conference 2010 (2010)
4. Canino, D., De Floriani, L., Weiss, K.: IA*: An adjacency-based representation for non-manifold simplicial shapes in arbitrary dimensions. Computers & Graphics **35**(3), 747–753 (2011)
5. Čomić, L., De Floriani, L., Iuricich, F.: Modeling three-dimensional morse and morse-smale complexes. In: M. Breu, A. Bruckstein, P. Maragos (eds.) Innovations for Shape Analysis, Mathematics and Visualization, pp. 3–34. Springer Berlin Heidelberg (2013)
6. Čomić, L., De Floriani, L., Iuricich, F., Fugacci, U.: Topological modifications and hierarchical representation of cell complexes in arbitrary dimensions. Computer Vision and Image Understanding **121**, 2–12 (2014)
7. Čomić, L., Floriani, L.D.: Dimension-independent simplification and refinement of morse complexes. Graphical Models **73**(5), 261–285 (2011)
8. Čomić, L., Floriani, L.D., Iuricich, F.: Simplifying morphological representations of 2d and 3d scalar fields. In: I.F. Cruz, D. Agrawal, C.S. Jensen, E. Ofek, E. Tanin (eds.) GIS, pp. 437–440. ACM (2011)
9. Čomić, L., Floriani, L.D., Iuricich, F.: Multi-resolution cell complexes based on homology-preserving euler operators. In: R. González-Díaz, M.J. Jiménez, B. Medrano (eds.) DGCI, *Lecture Notes in Computer Science*, vol. 7749, pp. 323–334. Springer (2013)
10. Čomić, L., Floriani, L.D., Iuricich, F.: Simplification operators on a dimension-independent graph-based representation of morse complexes. In: C.L.L. Hendriks, G. Borgefors, R. Strand (eds.) ISMM, *Lecture Notes in Computer Science*, vol. 7883, pp. 13–24. Springer (2013)
11. De Floriani, L., Fellegara, R., Iuricich, F., Weiss, K.: A spatial approach to morphological feature extraction from irregularly sampled scalar fields. In: Proceedings of the Third ACM SIGSPATIAL International Workshop on GeoStreaming, pp. 40–47. ACM (2012)
12. De Floriani, L., Mesmoudi, M.M., Morando, F., Puppo, E.: Decomposing non-manifold objects in arbitrary dimensions. Graphical Models **65**(1), 2–22 (2003)
13. De Floriani, L., Hui, A., Panozzo, D., Canino, D.: A Dimension-Independent Data Structure for Simplicial Complexes. In: S. Shontz (ed.) Proceedings of the 19^{th} International Meshing Roundtable (IMR 2010), pp. 403–420. Springer, Chattanooga, TN, USA (2010)
14. Floriani, L.D., Greenfieldboyce, D., Hui, A.: A data structure for non-manifold simplicial d-complexes. In: J.D. Boissonnat, P. Alliez (eds.) Symposium on Geometry Processing, *ACM International Conference Proceeding Series*, vol. 71, pp. 83–92. Eurographics Association (2004)
15. Floriani, L.D., Hui, A.: A scalable data structure for three-dimensional non-manifold objects. In: L. Kobbelt, P. Schröder, H. Hoppe (eds.) Symposium on Geometry Processing, *ACM International Conference Proceeding Series*, vol. 43, pp. 72–82. Eurographics Association (2003)
16. Floriani, L.D., Magillo, P., Puppo, E., Sobrero, D.: A multi-resolution topological representation for non-manifold meshes. In: Symposium on Solid Modeling and Applications, pp. 159–170 (2002)
17. Forman, R.: Morse theory for cell complexes. Advances in Mathematics **134**, 90–145 (1998)
18. Livesu, M., Usai, F., Puppo, E., Tarini, M., Scateni, R.: Coarse quad layouts from curve-skeletons (2014). Submitted for publication
19. Marcias, G., Pietroni, N., Panozzo, D., Puppo, E., Sorkine-Hornung, O.: Animation-aware quadrangulation. Computer Graphics Forum (EUROGRAPHICS/ACM SIGGRAPH Symposium on Geometry Processing) **32**(5), 167–175 (2013)
20. Marcias, G., Takayama, K., Pietroni, N., Panozzo, D., Puppo, E., Sorkine-Hornung, O., Cignoni, P.: Learning quadrangulations (2014). Submitted for publication
21. Matsumoto, Y.: An introduction to morse theory. translations of mathematical monographs, vol. 208. American Mathematical Society (2002)
22. Milnor, J.: Morse Theory. Princeton University Press, New Jersey (1963)
23. Munkres, J.: Elements of Algebraic Topology. Advanced book classics. Perseus Books (1984)
24. Panozzo, D., Lipman, Y., Puppo, E., Zorin, D.: Fields on symmetric surfaces. ACM Trans. Graph. (SIGGRAPH 2012) **31**(4), 111:1–111:12 (2012)
25. Panozzo, D., Puppo, E., Tarini, M., Pietroni, N., Cignoni, P.: Automatic construction of adaptive quad-based subdivision surfaces using fitmaps. IEEE Transaction on Visualization and Computer Graphics **17**(10), 1510–1520 (2011)
26. Panozzo, D., Puppo, E., Tarini, M., Sorkine-Hornung, O.: Frame fields: Anisotropic and non-orthogonal cross fields. ACM Trans. Graph. (SIGGRAPH 2014) (2014). To appear
27. Rocca, L., Puppo, E.: A virtually continuous representation of the deep structure of scale-space. In: A. Petrosino (ed.) Image Analysis and Processing ICIAP 2013, *Lecture Notes in Computer Science*, vol. 8157, pp. 522–531. Springer Berlin Heidelberg (2013)
28. Tarini, M., Pietroni, N., Cignoni, P., Panozzo, D., Puppo, E.: Practical quad mesh simplification. Comput. Graph. Forum (EUROGRAPHICS 2012) **29**(2), 407–418 (2010)
29. Tarini, M., Puppo, E., Panozzo, D., Pietroni, N., Cignoni, P.: Simple quad domains for field aligned mesh parametrization. ACM Trans. Graph. (SIGGRAPH Asia 2011) **30**(6) (2011)
30. Weiss, K., Fellegara, R., De Floriani, L., Velloso, M.: The PR-star octree: A spatio-topological data structure for tetrahedral meshes. In: Proceedings ACM SIGSPATIAL GIS, GIS '11. ACM (2011)
31. Weiss, K., Iuricich, F., Fellegara, R., De Floriani, L.: A primal/dual representation for discrete morse complexes on tetrahedral meshes. In: Computer Graphics Forum, vol. 32, pp. 361–370. Wiley Online Library (2013)